Real-Time Programming

Week 4: Signals, Sampling, Noise and Filters

Or: Why the wagon wheels go backwards on TV.

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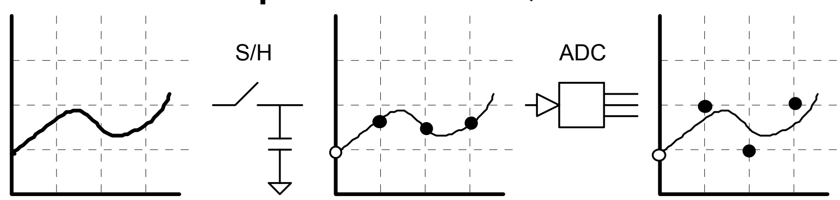
Objectives

- Continuous vs. Discrete
  - Time and Amplitude
- Signal Conversion:
  - Digital to Analog (D/A or DAC)
  - Analog to Digital (A/D or ADC)
- Number Systems
- Noise, Errors and Filters

Continuous vs. Discrete



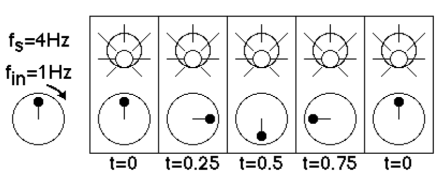
Continuous Time and Amplitude

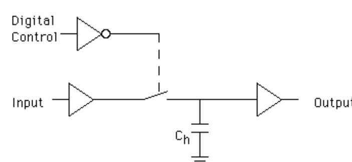
Discrete Time and Continuous Amplitude

Discrete Time and Discrete Amplitude

Sampled Signals

- Samples
- Ideally Instantaneous
- Strobe Light Flashing
  - Strobe Illuminates the Spot on a Rotating Disk:
- Sampling Yields
  - Discrete Samples
  - A Subset of Reality
  - But Still Continuous in Amplitude



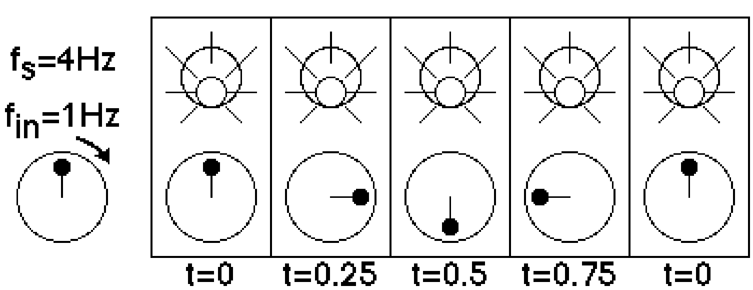


Sampling - Nyquist

- Sampling Frequency = f_s
- Input Frequency = f_{in}
- **Nyquist Criteria:** $f_s >= 2 * f_{in}$
  - $f_s < 2 * f_{in}$ results in **Aliasing**
  - "Nyquist Frequency" = $f_s/2$
- Aliasing is when $f_{apparent} < f_{in}$
- Strobe & Spinning Disk Analogy

Sampling - Strobe Analogy

- Disk Rotates Once/second, Light Flashes 4/sec.
- Four samples per rotation:
  - fs > 2*fin, so it meets Nyquist Criteria => No Aliasing



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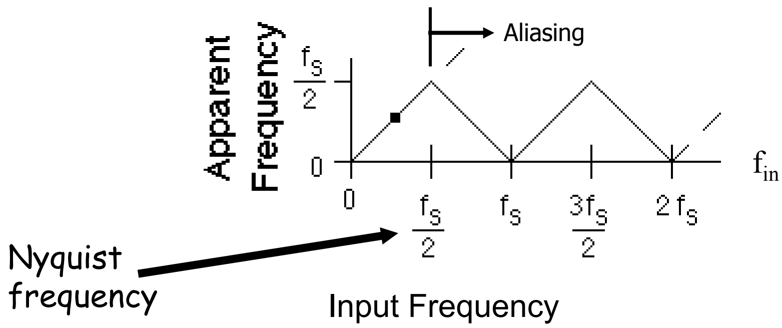
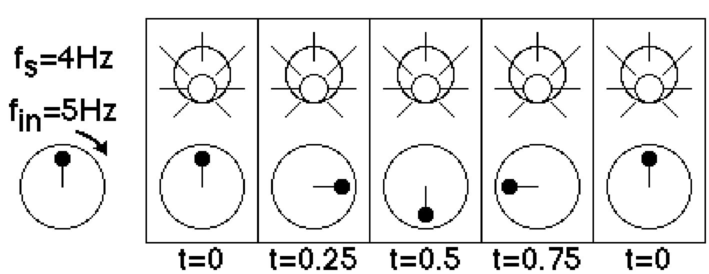
Apparent Frequency

- Apparent Frequency = Input Signal Frequency
- **<u>IF</u>** Nyquist Criteria is Satisfied ($f_{in} <= f_s/2$)



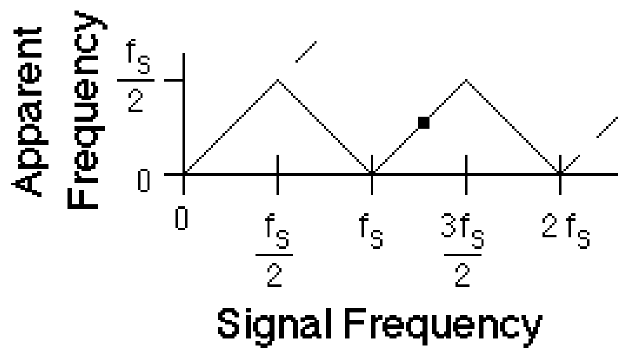
Aliasing

- Aliasing Occurs When $f_{in} > f_s/2$
- Apparent Frequency is 1 Hz, not 5 Hz!

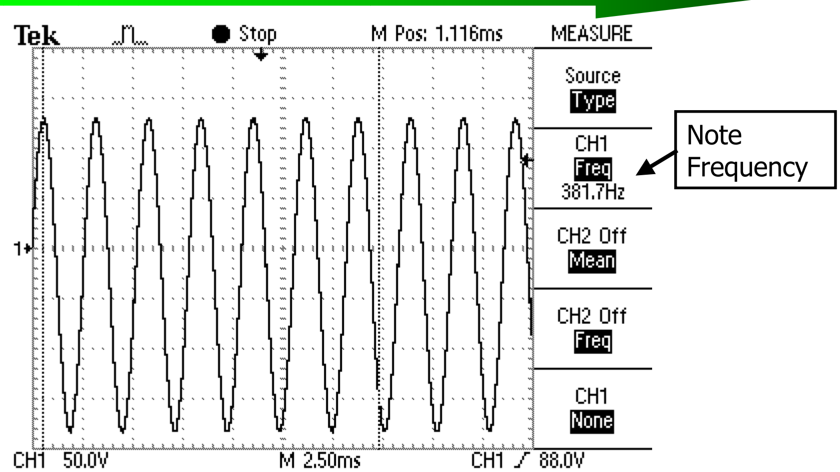


Alias Frequency

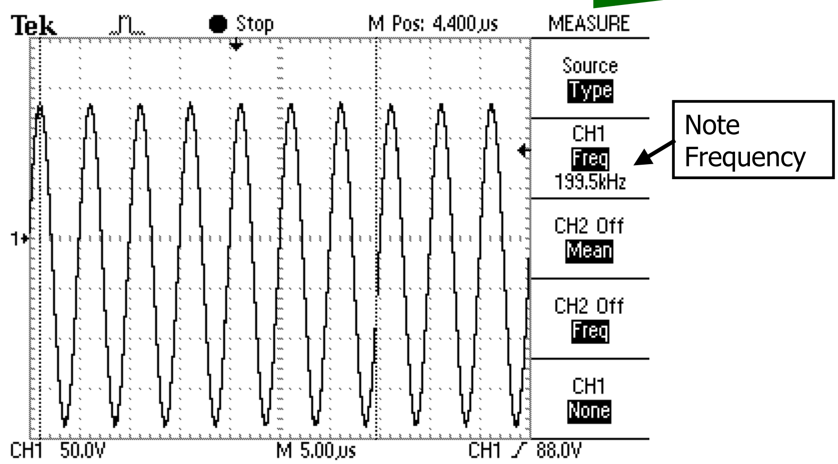
- Input Frequency is 5 Hz, Sampling at 4 Hz
- Alias is Indistinguishable From a 1 Hz Input!



Scope screen shot 1



Scope screen shot 2

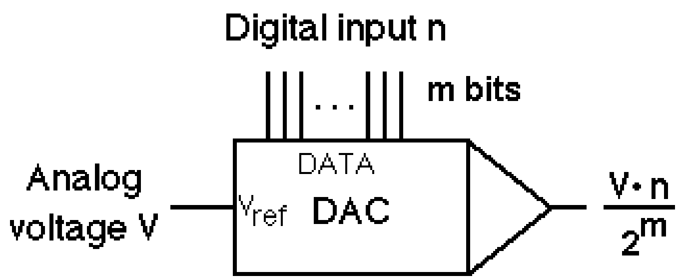


Aliasing

- Same input signal for both screen shots
  - Automatic frequency measurement
    - Screen1: f = 381 Hz at 2.5 mS/div
    - Screen2: f = 199 kHz at 5 uS/div
  - Actual Signal frequency = 200 kHz !
- Screen 1 shows effects of aliasing in a digital oscilloscope display

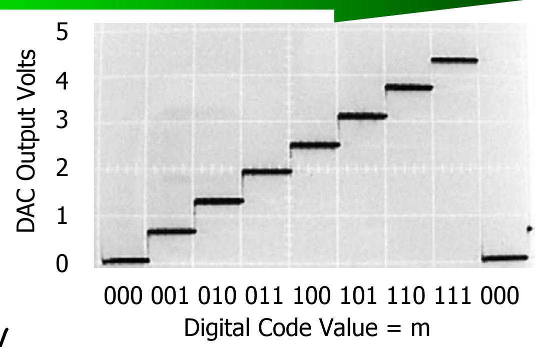
Generic D/A Converter

- Output is a Fraction Of the Reference Input
  fraction = n / (2^m) where n ranges from 0..2^m-1
- 8-bit DAC output = (0 to 255/256) * V



Ideal 3 Bit DAC Output

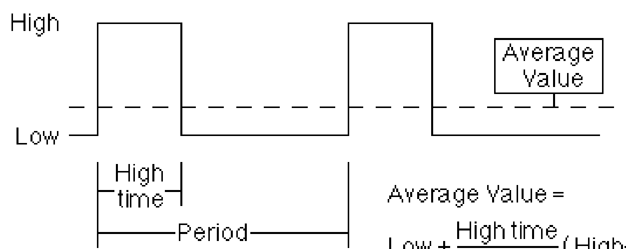
- Output V_{DAC} = $(m/2^n) * V_{ref}$
- Example:
  - V_{ref} = 5 V
  - n = 3
  - m=000..111
  - V_{DAC} = (m/8)*5 V



DAC Output is a fraction of reference proportional to the magnitude of the code

PWM DAC

- Pulse Width Modulation
  - *Average Value is Desired Output*
  - *Allows Use of Digital Output for Analog Signal*



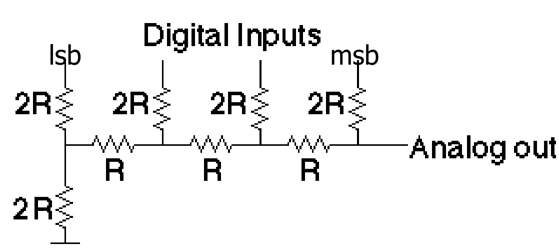
Average Value =

$$Low + \frac{High\ time}{Period}(High - Low)$$

DAC R-2R Network

- R-2R Weighted Resistor network
- Inputs Switched Between V_{ref} and Ground
- Output a Fraction of V_{ref} Proportional to Code

- Practical
- Easy to Make Precision R's with Values of R and 2*R



D/A Converter Types

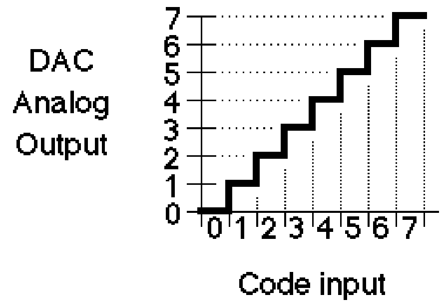
- Resistor Networks
- Current Sources
- Voltage and Current Output
- Other Types
  - Capacitor Networks
  - Pulse Width Modulation
  - Delta-Sigma

D/A Converter Types

- Unipolar (Unsigned)
- Bipolar (Signed)
  - Offset Binary
  - Signed Magnitude
  - Two's Complement

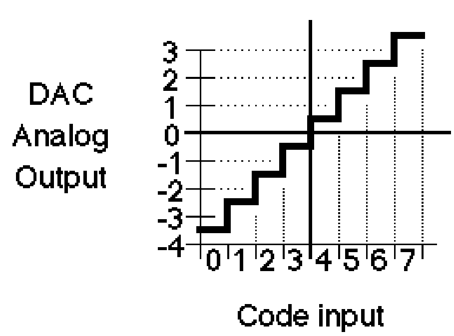
Unipolar DAC

- Unsigned Binary
  - 3-Bit DAC Shown
  - Ideal (No Errors)
- DAC Out =
  - Reference * $m/2^n$
    - Input=m
    - n bit DAC
  - Range: 0 to 2^n-1
  - Resolution: 1/ 2^n



Offset Binary DAC

- Unipolar DAC Shifted Negative ~1/2 Range
- Example: 3-bit Shown
  - Same as Unipolar, but
  - Shifted Down 3.5 V
  - Range: -3.5 to +3.5
  - Resolution: 1
  - No True Zero Output

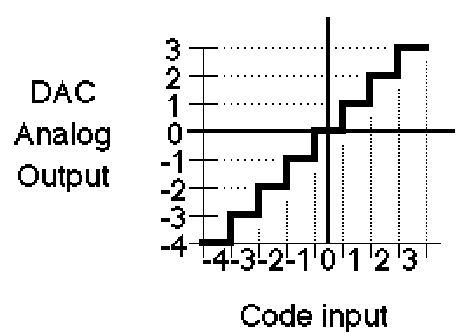


Signed Magnitude DAC

- Unipolar DAC plus Analog Circuitry to:
  - Multiply DAC Output by +1 or -1 Depending on the Sign Bit Input
  - More Complex Than Offset Binary
  - Equivalent to n+1 bit signed DAC
  - Symmetrical about Zero: +0 and -0
- Range: $-Vref*(2^n-1)/2^n$ to $+Vref*(2^n-1)/2^n$
- Resolution: $1/2^n$

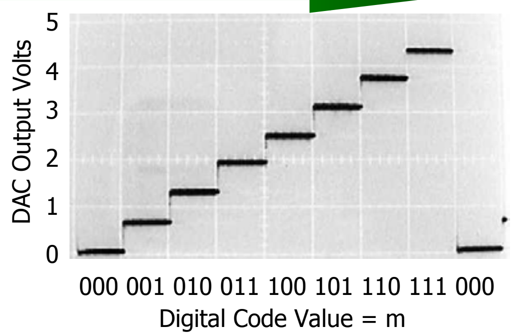
Two's Complement DAC

- Code MSB = -2^n
- Example: 3-bit
  - Range: -4 to +3
  - Resolution: 1
  - Reference = 4
- True Zero Output
- Asymmetrical Output About Zero
- Simplest Software



DAC Range, Resolution

- $V_{DAC} = V_{ref}\,(m/2^n)$
- Range:
  - 0 to V_{ref} - 1LSB
  - 0 to 5(7/8)v
- Resolution = lsb:
  - $V_{res} = V_{ref}\,(1/2^n)$
  - 1 LSB = 5(1/8)v



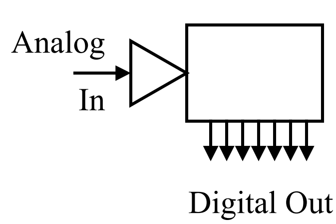
Range is 0v (code 000) to 4.325v (111)

Resolution is 0.625v (001) = 1 LSB = (5/8)v

Quantization

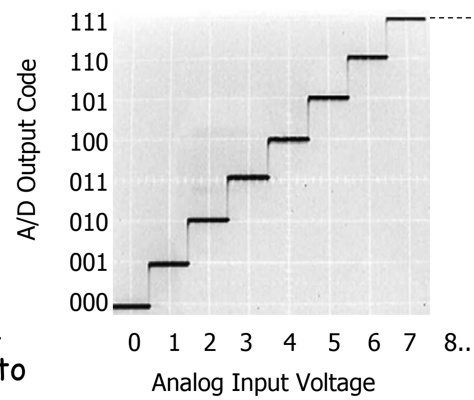
- Converting
  - Continuous Input to a Number
- Analog to Digital
  - A/D or ADC
  - Analog Input
  - Digital Output

A/D Converter



Ideal 3 Bit A/D (Quantizer)

- Output number
  $m = (V_{in}/V_{ref})2^n$
- Example:
  - V_{ref} = 8 V
  - V_{in} = 0..8 V
  - m=000..111
- A/D Output is a code number proportional to the input magnitude

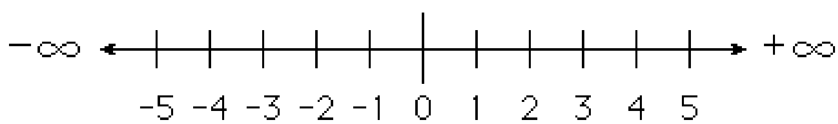


Range, Resolution & Accuracy

- Range: Extremes that can be represented
  - ex: -32768 to +32767, -5.12V to +5.11V
- Resolution: Number of Bits or the Smallest Difference that can be Represented
  - ex: 1 mV, 1 LSB, or 0.1% of Full Scale
- Accuracy: How Close is Converted Value Relative to (deviates from) Correct Value
  - ex: +/-2 LSBs, 1/2 LSB, 1% of Full Scale

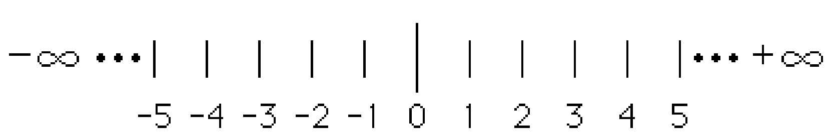
Real Number System

- Analog (Continuous) Systems
- Real Number System
  - The Real World!
  - Infinite Range and Resolution



Integer Number Systems

- Digital Systems -Discrete Numbers
  - Unsigned Binary
  - Signed Binary
    - Offset
    - Two's Complement
    - Fixed Point



Digital Number Systems

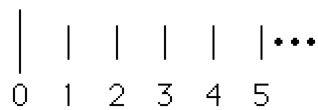
- Digital Approximations of Real Values
- Finite Range and Resolution
- Unsigned Numbers
- Signed Numbers
  - Two's Complement
- Fixed Point Numbers
  - Fractional Values
- Floating Point: variable range and resolution

Unsigned vs. Signed Nos.

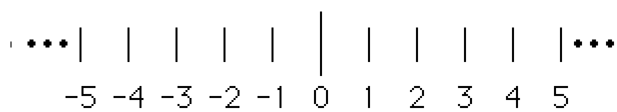
- Unsigned Number Range:
  - n bits Represent 2^n Integers from 0 .. 2^n -1
  - In Converter Speak: A Unipolar A/D or D/A
    - Meaning one direction (positive)
  - Unsigned Integer Values
- Signed Number (a.k.a. Bipolar) Range
  - n bits Represent 2^n Signed Integers
  - Range: $-2^{n/2}$ .. $+2^{n/2}$ -1

Number Lines

- Unsigned Integer Number Line:



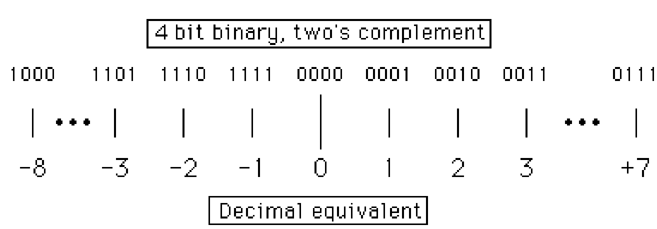
- Signed Integer Number Line:



Example: 4 Bit 2's Comp.

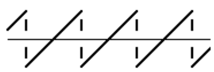
- Ex: $1011_2 = -5_{10}$
- MSB is Negative
- Range: -8 to +7
- Resolution: 1

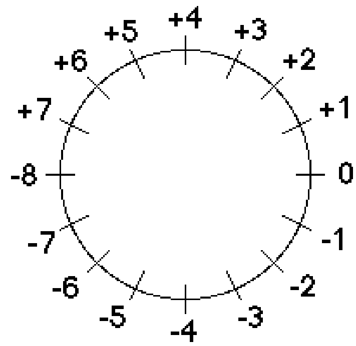
-2^3	2^2	2^1	2^0		
-8	4	2	1		
1	0	1	1_2		
-8 +	0 +	2 +	1	=	-5_{10}



Number Circle

- Limited Range:
  - For n bits:
  - $-2^{n/2}$ .. $2^{n/2}-1$
- Overflow:
  - 7+1 = - 8
  - -8 -1 = +7





Twos complement number circle

Fixed Point Numbers

- Binary Point Placed to the Left of the LSB
- Value is Scaled by a Negative Power of 2

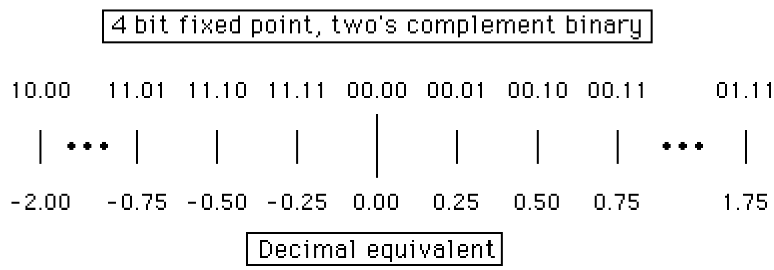
Binary Point

-2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	2^{-4}
-8	4	2	1		1/2	1/4	1/8	1/16
1	0	1	1	.	1	0	0	1_2

-8 + 0 + 2 + 1 + 0.5 + 0 + 0 + 0.0625 $= -4.4375_{10}$

Example: 4 bit Fixed Point

- 4 bit 2's Complement Numbers: x x . x x
  - Range: -2.00 to +1.75 Resolution: 0.25
  - Scale Factor: 4



Unipolar Converter Format

- For a 16 Bit Unipolar Converter
  - Format: 0.xxxx xxxx xxxx xxxx
  - Output Codes: 0 .. $2^{16}-1$ or 0 .. 65,535
  - Input Range: 0 .. Vref * $(2^{16}-1) / 2^{16}$
- Example:
  - .1100 0000 0000 0000 = 0.75 * Vref
  - Or 7.5 V if Vref = 10 V

Bipolar Converter Format

- Ex: 16 Bit 2's Complement Converter
  - x.xxx xxxx xxxx xxxx -- scale factor: 32,768
  - Range: -1*Vref to +1(-1LSB)*Vref
  - Resolution: 1 part in 2^{15} or Vref/32,768
- Ex: 4 Bit 2's Complement Converter
  - x.xxx scale factor: 8
  - Range -1*Vref to +(7/8)*Vref
  - Resolution (1/8)*Vref

Dynamic Range

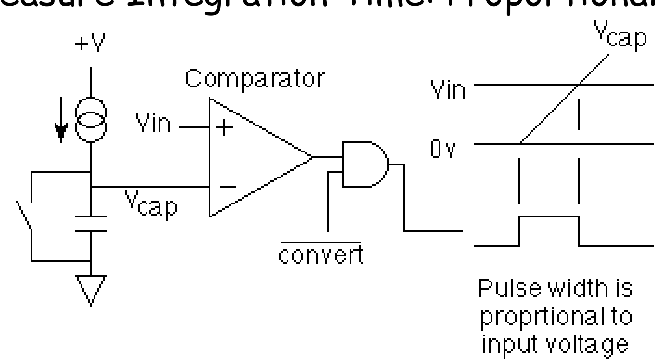
- The Dynamic Range of a Converter is the Ratio of the Largest to the Smallest Value: Full Scale to the LSB
- Dynamic Range is factor of two for each bit of resolution, since bit weight is 2^n
- Ex: For an 8 bit converter, range is 256, resolution is 1: dynamic range is 256:1 or 20 log (256/1) dB = 6*8 dB = 48 dB

A/D Converter Output

- For a Unipolar (Unsigned) A/D:
  - Unipolar means Range is 0 to Maximum
  - n bit resolution => range 0 to $(2^n-1)/ 2^n$ * Vref
  - Resolution of n bit A/D = Vref * $1/2^n$
  - Bipolar (signed) means -Min to +Max
- Output is a number, Q, the quantized Vin
  - Q is the Vin represented as a Fraction of Vref
- The A/D output number is a fraction of Vref

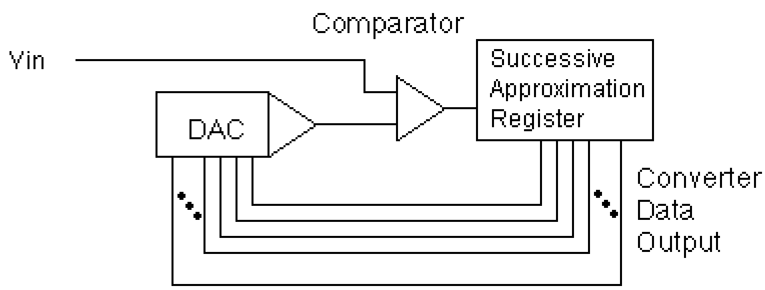
Integrating A/D

- Integrate a Constant; Compare to Vin
- Measure Integration Time: Proportional to Vin



Successive Approximation

- MSB = 1 if Vin > Vref/2, Zero Otherwise
- Next to MSB = 1 if Vin > Vref/4 and so on...



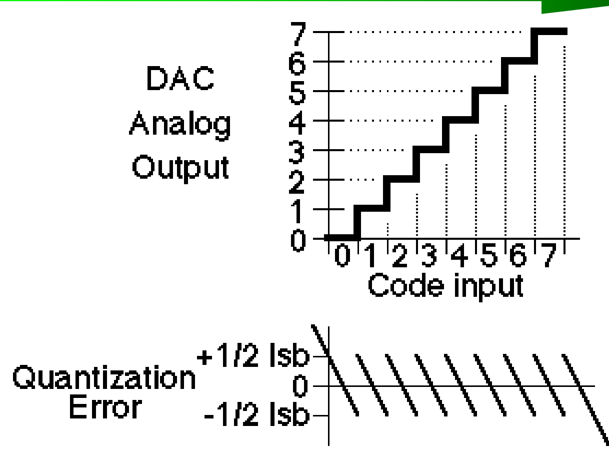
Serial Successive Approx.

- Most Common for Microcontroller Apps
- Simple Interface, Small Package
- Comparator Output Clocked Out Serially
- Resolutions from 8 to 16 Bits
- Simple, Low Cost Solution
- Reduced Pin Count

Accuracy vs. Resolution

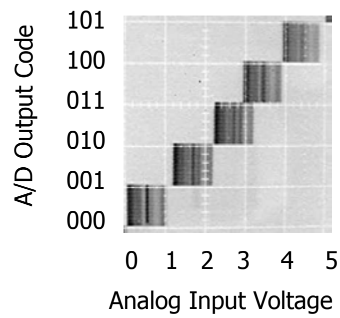
- Resolution: the number of converter bits
  - Or the "Quantum" value:
    - Quantized Amplitude Equivalent to One LSB
    - Smallest Measurable Difference
- Accuracy:
  - How close the actual value is to the ideal one
  - Usually specified as the maximum deviation:
    - Maximum Error as a % of Full Scale

D/A-A/D Quantization Error



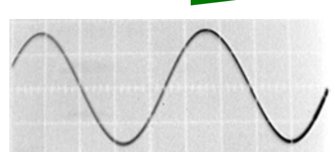
Real 3 Bit A/D Output

- Output code changes due to input noise
- Transition point is not exactly half way between ideal inputs

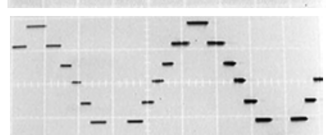


A/D Quantization

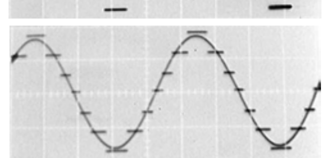
- Analog Sine Input



- Quantized Sine
  - Estimate of input

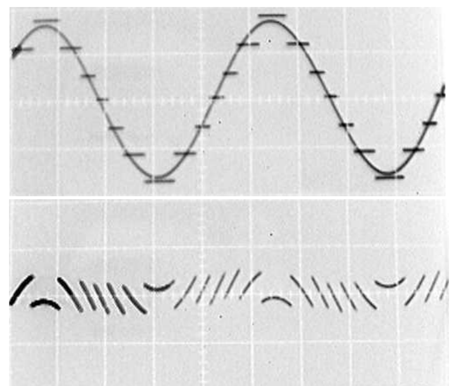


- Analog and Quantized signals overlaid



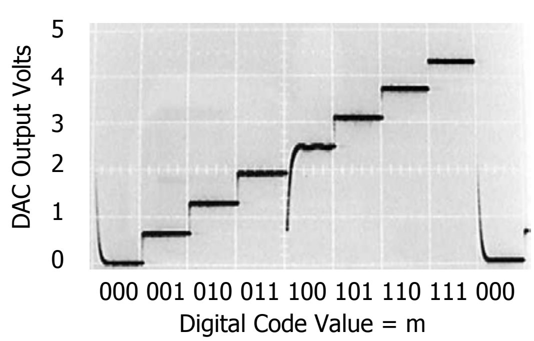
A/D Quantization Error

- Estimate is Imperfect
  - A perfect Quantizer can only pick the nearest value
- Quantization Error:
  - Difference between the analog value and quantizer estimate



DAC Output Glitch

- Bits don't change at exactly the same time
- Ex: $T_f < T_r$
  - Results in
  - 011-000-100

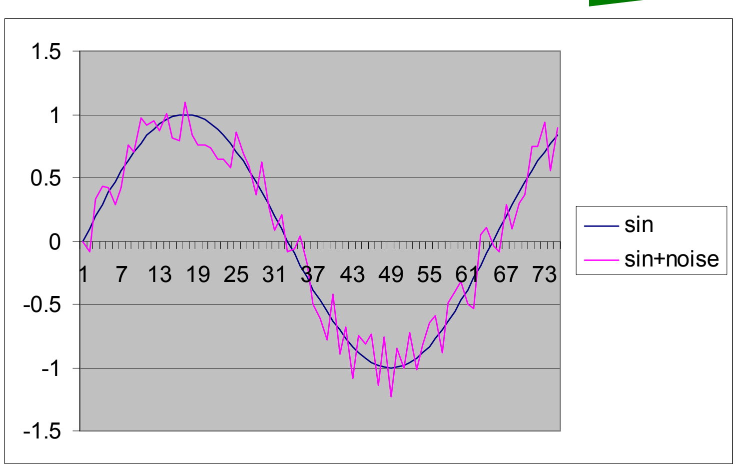


DAC Output exhibits glitch on MSB 0->1 transition between codes 011 and 100

Signal or Noise?

- What's the Difference?
- Signal == What We Want
- Noise == Everything Else!
- Filtering:
  - Estimating the Noise Free Signal
  - Estimate = (Noisy Signal) - Estimate of Noise

Signal + Noise



Filtering

- Depends on Assumptions about:
  - Signal Characteristics
  - Noise Characteristics
- Common Assumptions:
  - Noise has Average Value == 0
  - Signal has Smaller Bandwidth than Noise
  - Noise is Additive
- Average Multiple Measurements

Filter = Moving Average

- Signal vs. Noise Amplitude
- Signal vs. Noise Frequencies
- Average: Mean or Median or ?
- Mean = (Sum of n values of x)/n = $(\Sigma x)/n$
- The Hard Part: Which Values to Use?
  - The Last n Values, for Starters...
  - Average inputs or inputs and outputs?

Weighted Average Inputs

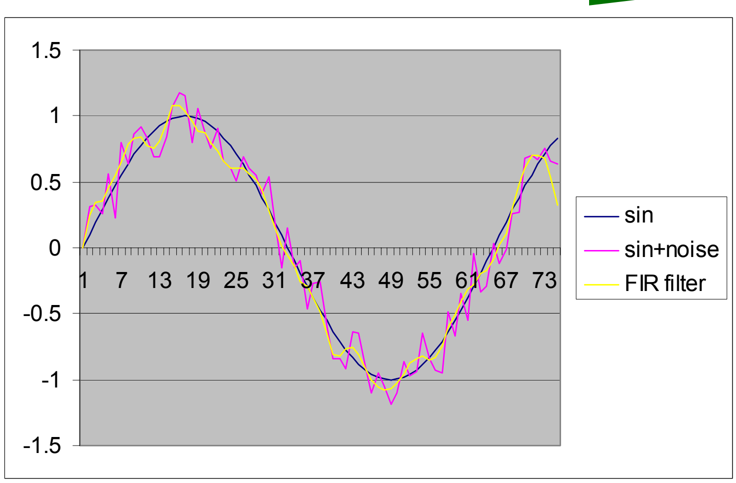
- Y_i = (Output) is the Average of
  - current (X_i) and past **inputs** ($X_{i-1}, X_{i-2}, X_{i-3}, \ldots$)
- Example, weighted average of inputs:

  $Y_i = C_0 * X_i + C_1 * X_{i-1}$
- Equally weighted:

  $Y_i = (X_i + X_{i-1}) / 2$

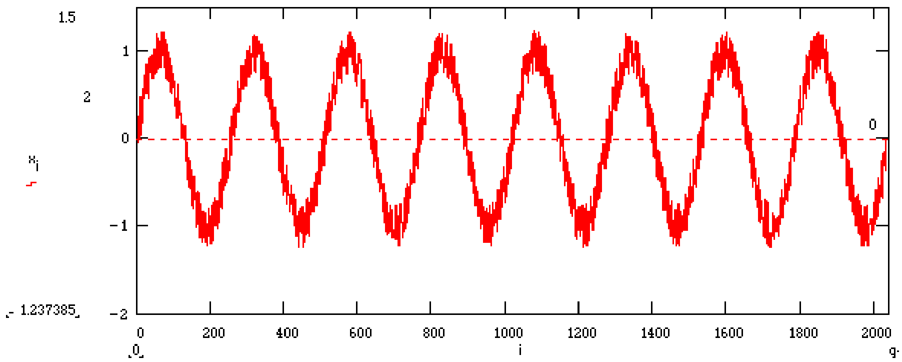
  $= 0.5 * X_i + 0.5 * X_{i-1}$
- So in this case, coefficient $C_0 = C_1 = 0.5$

Moving Average Filter



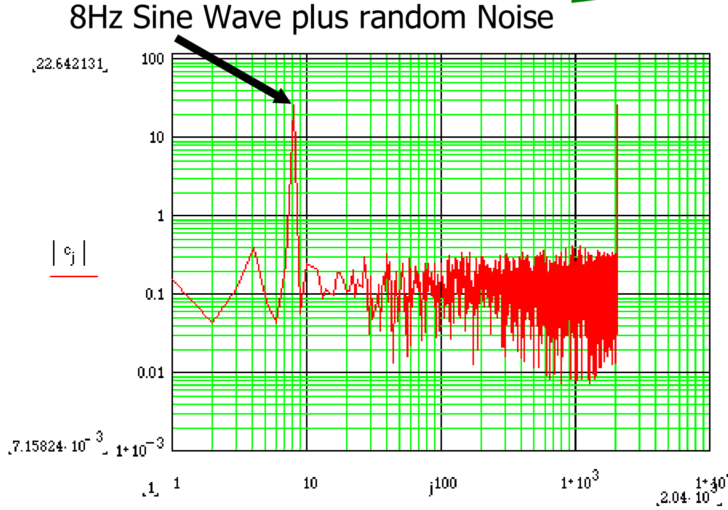
Amplitude vs. Time

8Hz Sine Wave + Noise



Amplitude vs. Frequency

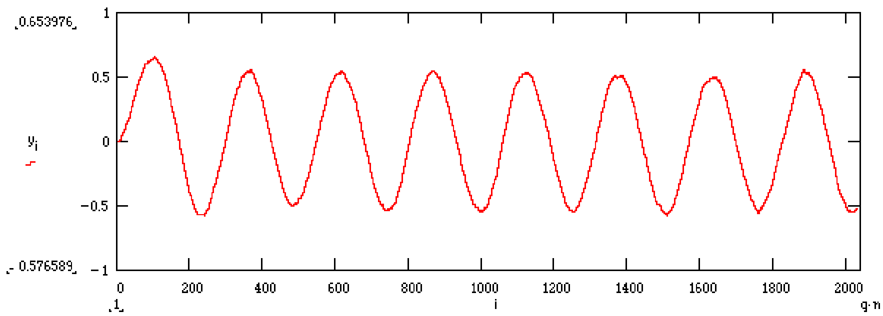
8Hz Sine Wave plus random Noise



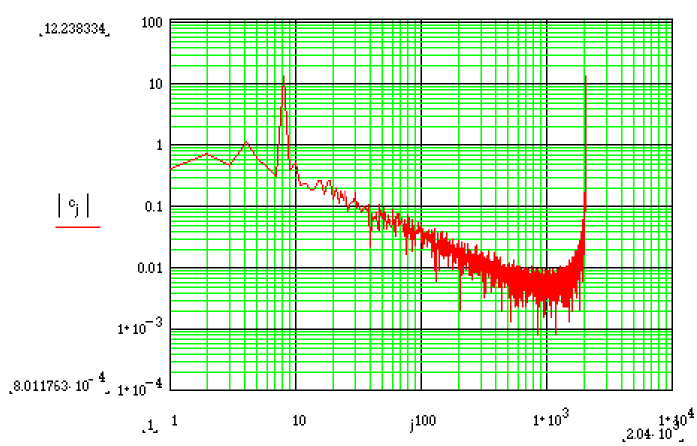
Filtered Output

- Filter Equation:

$$Y_i = (1/64) * X_i + (63/64) * Y_{i-1}$$



Filter Output vs. Frequency



Filter Terminology

- FIR – Finite Impulse Response
  - $Y_i = C_0X_i + C_1X_{i-1} + \ldots$
  - Non-recursive
- IIR – Infinite Impulse Response
  - $Y_i = C_0X_i + \ldots + C_1Y_{i-1} + \ldots$
  - Recursive

References

- Web page for filter design: http://www.nauticom.net/www/jdtaft/
- On ftp site, rtp/handouts:
  - filters.xls (Excel spreadsheet version)
  - filters.mcd (MathCAD version)
  - firiir.mcd (MathCAD version)

Summary

- Sampling, Nyquist, and Aliasing
- Conversion and Quantization
- Signals and Noise
- Filters